THE CONSEQUENCES OF ARROW'S THEOREM ON SYSTEMS ENGINEERING

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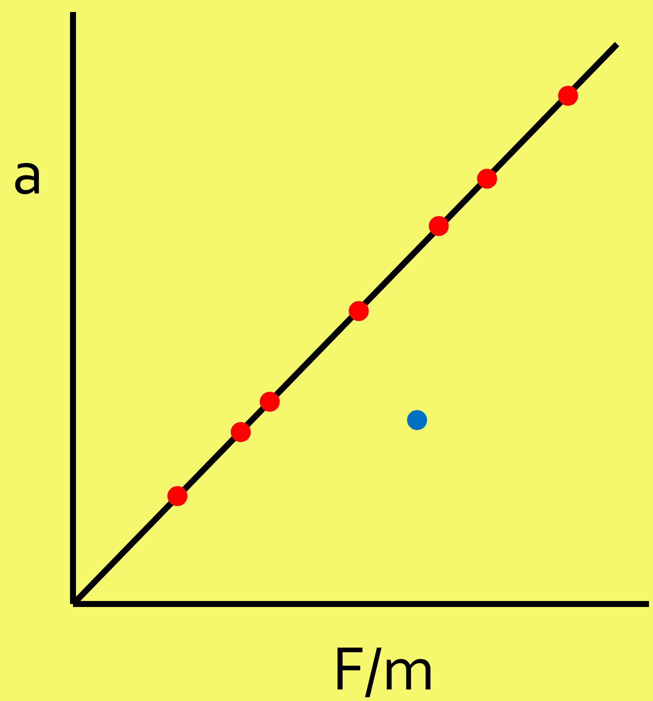
Backgroud

- What is science?
  - The search for fundamental laws of nature
  - Often framed as hypothesis testing
  - Example: We hypothesize that the acceleration of an object is proportional to the ratio of imposed force to mass
  - Approach: write a mathematical model, $a=F/m$, then compare model results to experimental evidence

Hypothesis testing



The model fills the space

Each data point is a point, n points fill nothing

One valid outlier disproves the hypothesis

Ergo, we only disprove hypotheses

Hypothesis testing

- We accept an hypothesis as true only after considerable testing fails to disprove it
- But—while we may accept an hypothesis as true, we never prove this
- Knowledge is advanced when the data provide such a compelling case that no respectable, knowledgeable person would disbelieve the hypothesis

Science vs. religion

- Science is based on disbelief—belief derives only when the evidence is sufficiently overwhelming and compelling that consensus is obtained
- Religion is based on belief—disbelief derives only when the evidence against the belief is overwhelming and compelling
- Science and religion are polar opposites

What is research?

- The process of finding out something we don't already know
- Scientific research has special characteristics
  - It is methodical—in advance of being done, it can be planned
  - It is repeatable—data are not random
  - It is verifiable—conclusions are based on tangible evidence

What is engineering?

- The deliberate manipulation of nature to benefit at least some segment of society
- Engineering is design
- Design is a process of decision making
- Decision making demands that we predict the future
- To predict the future, we accept that all the laws of nature apply everywhere all the time

What is engineering research?

- To do engineering, we must predict the future
- We want to predict the future in a manner that is consistent with what we believe
- All the laws of nature apply everywhere all the time in every engineered device
- But we cannot take into account all the laws of nature
- Question: Which laws of nature dominate the behavior of a particular device, and how can we represent them?

What is engineering research?

- Design is decision making
- Decision making is optimization
- All real decision making is under uncertainty and risk
  - Deterministic models are not predictive models
- Ergo, design is optimization under uncertainty and risk
- This isn't easy

Enter mathematics

- The goal of good design, i.e., good decision making, is consistency
- Mathematics is the “science” of consistency
- A mathematical framework is designed by stating a fundamental and self-consistent set of beliefs (axioms) and deriving theorems, which are the logical framework within which we work
- We use mathematics to enforce consistency

Bad math

Example 1

- $x=y$
  $x^2=xy$
  $x^2-y^2=xy-y^2$
  $(x+y)(x-y)=y(x-y)$
  $x+y=y$
  $2y=y$
  $2=1$
  Ergo, all numbers are equal

Example 2

- Solve for the largest positive integer

  For every positive integer, n, there is n^2, which is also a positive integer
  If n is the largest integer,
  $n \geq n^2$
  Hence, the largest positive integer is 1

What is systems engineering?

- A holistic view of the design process
- A life cycle approach to design
- Treating design as a decision-making process
- Design of big or complicated things
- The design of things that take more than one person to design

What is the goal of sys-e?

- To do design rationally, i.e., in a totally self-consistent manner—optimally
- Consistency in design means consistency in design decision making—i.e., decisions are consistent with:
  - Available alternatives
  - Beliefs on the performance of the alternatives
  - Preferences—what we want
- Remember what happens otherwise!

An aside

- Engineers are problem solvers
- What is a problem? The things at the end of each chapter in textbooks? Given x, find y?
  - We “solve” problems
  - We get answers
  - Answers are right or wrong
  - Answers are right if they are consistent with laws of nature, data or boundary conditions and the appropriate mathematics

Decisions are different

- We "make" a decision
  - Decisions are good or bad, not right or wrong
  - We get an outcome, not an answer
  - A good decision is one that is consistent with:
    - Available alternatives
    - Beliefs on the outcomes of the alternatives
    - Preferences over the outcomes
  - Decisions always involve the allocation of resources
- Alternatives and preferences are not part of problem solving, but decisions cannot be made in their absence

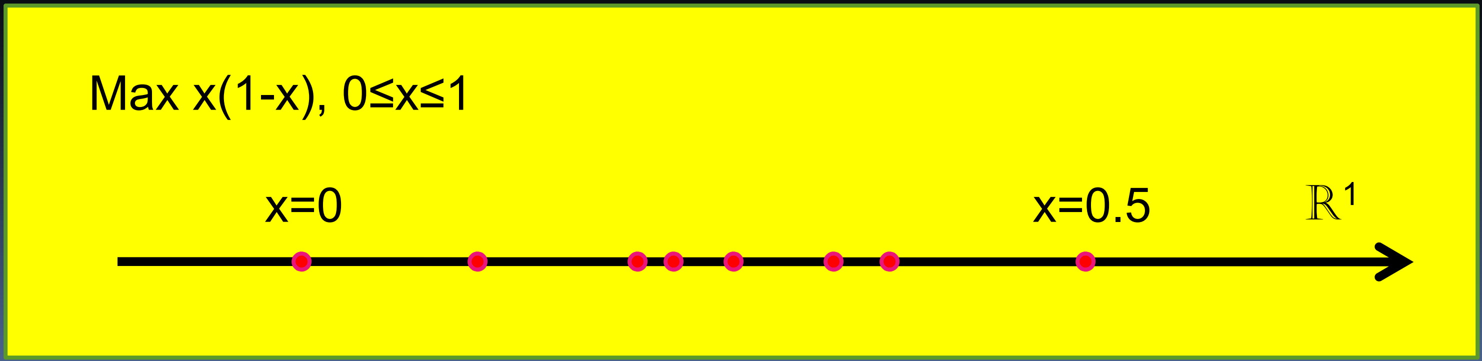
Decision makers make the big bucks

- Doctor vs. nurse
- Lawyer vs. paralegal
- Airline pilot vs. copilot
- Manager vs. worker

We need to be teaching engineering as a process of decision making, not problem solving

What is optimization?

- Optimization is doing the best that you can as measured against a preference
- To optimize, we define alternatives, determine their outcomes, assign a value to each outcome, map the outcomes on the real number line and choose the rightmost point



When is optimization possible?

- The objective function must be a preference
- The objective function must exist
- The objective function must rank-order all outcomes in precisely the same order as would the decision maker
- In order that the objective function exist:
  - one and only one of the following must apply
    x>y, x>y, or x~y ordering must exist
  - If x>y and y>z, then x>z ordering is complete

Optima don't have to exist

- Smithers, go get us a pizza!
  - Monty A>E>P, A>P
  - Homer E>P>A, E>A
  - Larry P>A>E, P>E
- What pie does Smithers get?
  - A>E
  - E>P
  - P>A

No matter what pie Smithers chooses, there is a better one—there is no optimal pie

What’s this got to do with Arrow’s theorem?

- Arrow’s theorem is an aggregation theorem
- Four principles for any aggregation—e.g., mapping a set of preferences onto the real number line
  - x is unanimously preferred to y, z, etc., choose x
  - x is preferred to y and y is preferred to z, choose x over z
  - the choice between x and y does not depend on the existence of z, w, etc.
  - the choice is not made by a “dictator”

Arrow’s impossibility theorem

- Any aggregation algorithm that respects the first three conditions is necessarily a dictatorship
- Consequences:
  - Optima do not necessarily exist
  - There is no mathematic that assures consistency with multiple decision makers
  - Systems engineering is a matter of damage control

Little rubber balls

- Alternative colors: R, G, O, Y, B
- Survey 100 kids
  - 45: RBYOG
  - 25: GBYOR
  - 17: OBYGR
  - 13: YBOGR

Hey kids, what's your favorite color ball?

1 kid, 1 vote

R – 45
G – 25
O – 17
Y – 13
B – 0

Preference order, RGOYB

Taste test

- Which flavor do you like best?
  - 60: N>C>P
  - 50: P>C>N
  - 40: C>P>N
- By vote: N>P>C, ergo produce N
- By pairwise comparison: C>P (100:50), P>N (90:60), C>N (90:60)—clearly C is the preferred flavor
- Coke, Pepsi, New Coke

GM customer focus centers

- Reuters, Jan. 21, 2002
  - GM closes its customer focus centers after producing the Pontiac Aztec
  - New process—new designs will be tested in customer clinics only after design teams produce final models

Pontiac Aztec, voted ugliest car in the world

Material selection

- Choose a material from alternatives A, B, C, D
- Tests:
  - T1: A C B D
  - T2: A C B D
  - T3: B A C D
  - T4: B A C D
  - T5: B A C D
- Score using Borda count

Test	A	B	C	D
T1	3	1	2	0
T2	3	1	2	0
T3	2	3	1	0
T4	2	3	1	0
T5	2	3	1	0
Totals	12	11	7	0

The best material is A

After the dream

- C is inferior to A, hence we don't test it
- Tests:
  - T1: A B D
  - T2: A B D
  - T3: B A D
  - T4: B A D
  - T5: B A D
- The test results are identical

Test	A	B	D
T1	2	1	0
T2	2	1	0
T3	1	2	0
T4	1	2	0
T5	1	2	0
Totals	7	8	0

The best material is B

Continuous improvement

- We have product S, can we improve it?
- Suggested changes A, B, C
- Possible products: S, S_A, S_B, S_C, S_{AB}, S_{AC}, S_{BC}, S_{ABC}
- Product improvement team:
  - Jan $S_{AB} > S_A > S > S_B > S_C > S_{AC} > S_{BC} > S_{ABC}$
  - Pat $S_A > S > S_B > S_C > S_{BC} > S_{AC} > S_{ABC} > S_{AB}$
  - Michael $S_B > S > S_C > S_{CB} > S_{AC} > S_{ABC} > S_{AB} > S_A$

Continuous improvement

- Given the team's preferences:
  - Jan $S_{AB} > S_A > S > S_B > S_C > S_{AC} > S_{BC} > S_{ABC}$
  - Pat $S_A > S > S_B > S_C > S_{BC} > S_{AC} > S_{ABC} > S_{AB}$
  - Michael $S_B > S > S_C > S_{CB} > S_{AC} > S_{ABC} > S_{AB} > S_A$
- $S_{ABC} > S_{AB} > S_A > S$
  - The team thus moves from S to S_A to S_{AB} to S_{ABC}
  - But everyone agrees that $S > S_{ABC}$
- Continuous improvement made the product worse

A tenure case

- Three assistant professors get student ratings
  - Marc>Raj>Jan 33 students
    Marc>Jan>Raj 0 students
    Raj>Marc>Jan 25 students
    Raj>Jan>Marc 14 students
    Jan>Marc>Raj 25 students
    Jan>Raj>Marc 17 students
- Scores (pick one): Jan 42, Raj 39, Marc 33
- Jan gets tenure

But Raj accepts another job

- Ratings for Marc and Jan only

Marc>Jan	33 students
Marc>Jan	0 students
Marc>Jan	25 students
Jan>Marc	14 students
Jan>Marc	25 students
Jan>Marc	17 students

- Scores (pick one): Jan 56, Marc 58
- Who should get tenure?

Market survey

Two new products, A and B are tested in two markets against the existing product

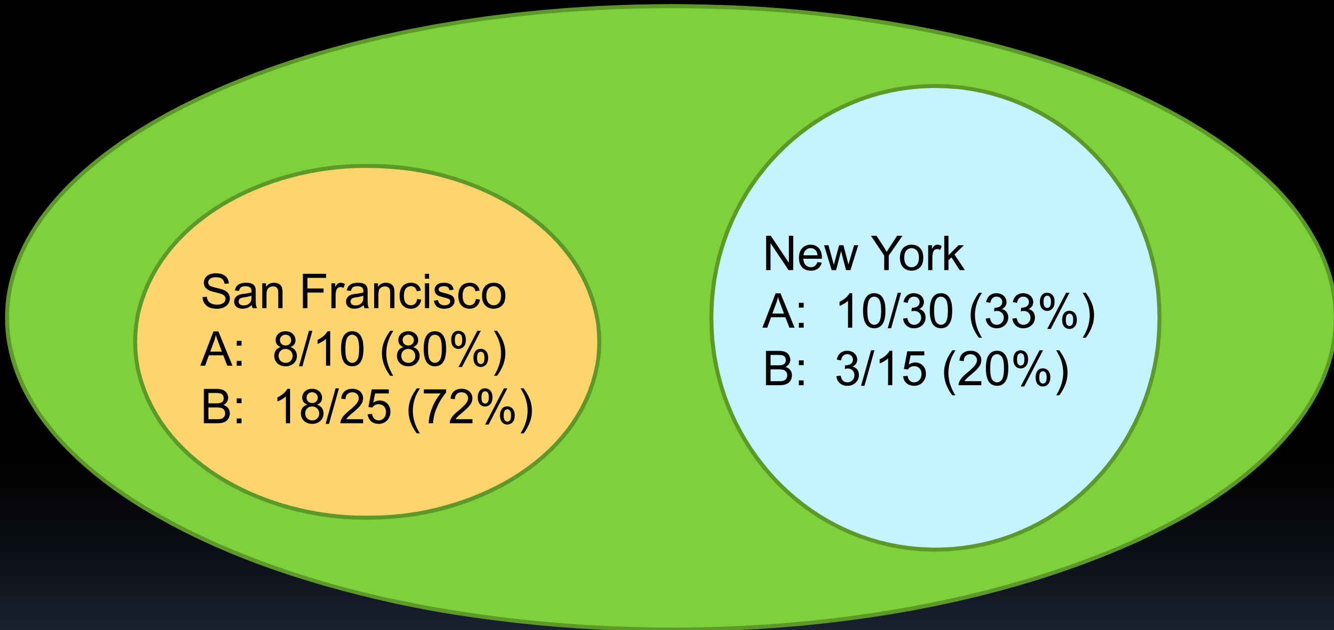
San Francisco
A: 8/10 (80%)
B: 18/25 (72%)

New York
A: 10/30 (33%)
B: 3/15 (20%)

A beats B in both cities

Market survey

We aggregate the results:

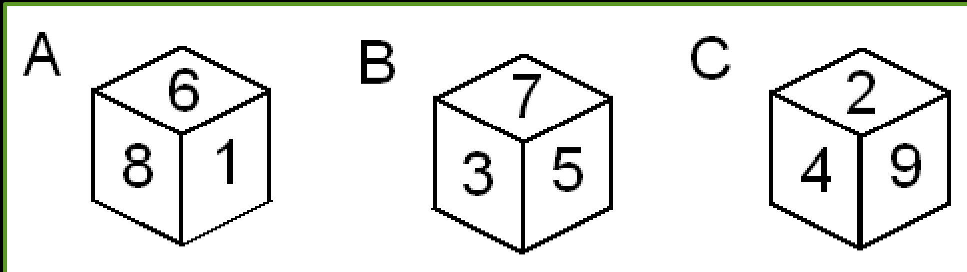


A is preferred 18 out of 40 times
B is preferred 21 out of 40 times
B is the better design

Loaded dice

- A game of chance: there are 3 dice, each with 3 numbers on the sides (the numbers repeat on opposite sides)
  - D_A: 1, 6, 8
  - D_B: 3, 5, 7
  - D_C: 2, 4, 9
- You choose a die, then I choose a die. We roll them and the person whose die lands with the highest number wins

You lose!



A vs. B		B vs. C		C vs. A	
1	3	3	2	2	1
1	5	3	4	2	6
1	7	3	9	2	8
6	3	5	2	4	1
6	5	5	4	4	6
6	7	5	9	4	8
8	3	7	2	9	1
8	5	7	4	9	6
8	7	7	9	9	8

A beats B
B beats C
C beats A

Black matter and such

- Engineering faux pas
  - Data fusion
  - Multi-scale modeling
  - Multi-objective optimization
  - QFD, Pugh, AHP
- Black matter—D. Saari suggests that there is much less (90% less) black matter than current models predict

So what?

- Seemingly good models might not be so good after all
- When rationality runs out, all bets are off—we go into a mode of damage control
- We lose the ability to do rational design at about the time we assign the second engineer to the project
- Really bad things can happen from here
- Is there hope anywhere?
  - Decision theory and Bayes theorem
  - Modeling
  - Mechanism design

Recommendations

- We need to take note of the mathematics of systems engineering
- Let’s take engineering education away from the scientists and start teaching engineering (decision making vs. problem solving)
- Stop teaching engineering as a deterministic application of scientific principles
- Stop teaching engineering students as though every one will become a professor
- Teach judgment