THE CONSEQUENCES OF ARROW’S THEOREM ON SYSTEMS ENGINEERING

George A. Hazelrigg
Deputy Division Director
NSF, CMMI Division
What is science?

- The search for fundamental laws of nature
- Often framed as hypothesis testing
- Example: We hypothesize that the acceleration of an object is proportional to the ratio of imposed force to mass
- Approach: write a mathematical model, $a=\frac{F}{m}$, then compare model results to experimental evidence
Hypothesis testing

The model fills the space

Each data point is a point, n points fill nothing

One valid outlier disproves the hypothesis

Ergo, we only disprove hypotheses
Hypothesis testing

- We accept an hypothesis as true only after considerable testing fails to disprove it
- But—while we may accept an hypothesis as true, we never prove this
- Knowledge is advanced when the data provide such a compelling case that no respectable, knowledgeable person would disbelieve the hypothesis
Science vs. religion

- Science is based on disbelieve—belief derives only when the evidence is sufficiently overwhelming and compelling that consensus is obtained.
- Religion is based on belief—disbelief derives only when the evidence against the belief is overwhelming and compelling.
- Science and religion are polar opposites.
What is research?

- The process of finding out something we don’t already know
- Scientific research has special characteristics
  - It is methodical—in advance of being done, it can be planned
  - It is repeatable—data are not random
  - It is verifiable—conclusions are based on tangible evidence
What is engineering?

- The deliberate manipulation of nature to benefit at least some segment of society
- Engineering is design
- Design is a process of decision making
- Decision making demands that we predict the future
- To predict the future, we accept that all the laws of nature apply everywhere all the time
What is engineering research?

- To do engineering, we must predict the future.
- We want to predict the future in a manner that is consistent with what we believe.
- All the laws of nature apply everywhere all the time in every engineered device.
- But we cannot take into account all the laws of nature.
- Question: Which laws of nature dominate the behavior of a particular device, and how can we represent them?
What is engineering research?

- Design is decision making
- Decision making is optimization
- All real decision making is under uncertainty and risk
  - Deterministic models are not predictive models
- Ergo, design is optimization under uncertainty and risk
- This isn’t easy
Enter mathematics

- The goal of good design, i.e., good decision making, is consistency
- Mathematics is the “science” of consistency
- A mathematical framework is designed by stating a fundamental and self-consistent set of beliefs (axioms) and deriving theorems, which are the logical framework within which we work
- We use mathematics to enforce consistency
Bad math

Example 1

- $x = y$
- $x^2 = xy$
- $x^2 - y^2 = xy - y^2$
- $(x+y)(x-y) = y(x-y)$
- $x+y = y$
- $2y = y$
- $2 = 1$

Ergo, all numbers are equal

Example 2

- Solve for the largest positive integer

For every positive integer, $n$, there is $n^2$, which is also a positive integer. If $n$ is the largest integer, $n \geq n^2$

Hence, the largest positive integer is 1
What is systems engineering?

- A holistic view of the design process
- A life cycle approach to design
- Treating design as a decision-making process
- Design of big or complicated things
- The design of things that take more than one person to design
What is the goal of sys-e?

- To do design rationally, i.e., in a totally self-consistent manner—optimally
- Consistency in design means consistency in design decision making—i.e., decisions are consistent with:
  - Available alternatives
  - Beliefs on the performance of the alternatives
  - Preferences—what we want
- Remember what happens otherwise!
An aside

- Engineers are problem solvers
- What is a problem? The things at the end of each chapter in textbooks? Given $x$, find $y$?
  - We “solve” problems
  - We get answers
  - Answers are right or wrong
  - Answers are right if they are consistent with laws of nature, data or boundary conditions and the appropriate mathematics
Decisions are different

- We “make” a decision
  - Decisions are good or bad, not right or wrong
  - We get an outcome, not an answer
  - A good decision is one that is consistent with:
    - Available alternatives
    - Beliefs on the outcomes of the alternatives
    - Preferences over the outcomes
  - Decisions always involve the allocation of resources

- Alternatives and preferences are not part of problem solving, but decisions cannot be made in their absence
Decision makers make the big bucks

- Doctor vs. nurse
- Lawyer vs. paralegal
- Airline pilot vs. copilot
- Manager vs. worker

We need to be teaching engineering as a process of decision making, not problem solving.
What is optimization?

- Optimization is doing the best that you can as measured against a preference.
- To optimize, we define alternatives, determine their outcomes, assign a value to each outcome, map the outcomes on the real number line and choose the rightmost point.

Max \( x(1-x), 0 \leq x \leq 1 \)

\[
\begin{align*}
\text{x=0} & \quad \text{x=0.5} & \quad \mathbb{R}^1
\end{align*}
\]
When is optimization possible?

- The objective function must be a preference
- The objective function must exist
- The objective function must rank-order all outcomes in precisely the same order as would the decision maker
- In order that the objective function exist:
  - one and only one of the following must apply
    - \( x>y, x>y, \) or \( x\sim y \) ordering must exist
  - If \( x>y \) and \( y>z \), then \( x>z \) ordering is complete
Optima don’t have to exist

- Smithers, go get us a pizza!
  - Monty        A>E>P, A>P
  - Homer        E>P>A, E>A
  - Larry        P>A>E, P>E

- What pie does Smithers get?
  - A>E
  - E>P
  - P>A

No matter what pie Smithers chooses, there is a better one—there is no optimal pie.
What’s this got to do with Arrow’s theorem?

- Arrow’s theorem is an aggregation theorem
- Four principles for any aggregation—e.g., mapping a set of preferences onto the real number line
  - x is unanimously preferred to y, z, etc., choose x
  - x is preferred to y and y is preferred to z, choose x over z
  - the choice between x and y does not depend on the existence of z, w, etc.
  - the choice is not made by a “dictator”
Arrow’s impossibility theorem

- Any aggregation algorithm that respects the first three conditions is necessarily a dictatorship

- Consequences:
  - Optima do not necessarily exist
  - There is no mathematic that assures consistency with multiple decision makers
  - Systems engineering is a matter of damage control
Little rubber balls

- Alternative colors: R, G, O, Y, B
- Survey 100 kids
  - 45: RBYOG
  - 25: GBYOR
  - 17: OBYGR
  - 13: YBOGR

Hey kids, what’s your favorite color ball?
1 kid, 1 vote
- R – 45
- G – 25
- O – 17
- Y – 13
- B – 0
Preference order, RGOYB
Taste test

- Which flavor do you like best?
  - 60: N>C>P
  - 50: P>C>N
  - 40: C>P>N

- By vote: N>P>C, ergo produce N
- By pairwise comparison: C>P (100:50), P>N (90:60), C>N (90:60)—clearly C is the preferred flavor

- Coke, Pepsi, New Coke
GM customer focus centers

- Reuters, Jan. 21, 2002
  - GM closes its customer focus centers after producing the Pontiac Aztec
  - New process—new designs will be tested in customer clinics only after design teams produce final models

Pontiac Aztec, voted ugliest car in the world
Material selection

- Choose a material from alternatives A, B, C, D
- Tests:
  - T1: A C B D
  - T2: A C B D
  - T3: B A C D
  - T4: B A C D
  - T5: B A C D
- Score using Borda count

<table>
<thead>
<tr>
<th>Test</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T5</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>12</td>
<td>11</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

The best material is A
After the dream

- C is inferior to A, hence we don’t test it
- Tests:
  - T1: A B D
  - T2: A B D
  - T3: B A D
  - T4: B A D
  - T5: B A D
- The test results are identical

<table>
<thead>
<tr>
<th>Test</th>
<th>A</th>
<th>B</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>T4</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>T5</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Totals 7 8 0

The best material is B
Continuous improvement

- We have product S, can we improve it?
- Suggested changes A, B, C
- Possible products: S, S_A, S_B, S_C, S_AB, S_AC, S_BC, S_ABC
- Product improvement team:
  - Jan  \( S_{AB} > S_A > S > S_B > S_C > S_{AC} > S_{BC} > S_{ABC} \)
  - Pat \( S_A > S > S_B > S_C > S_{BC} > S_{AC} > S_{ABC} > S_{AB} \)
  - Michael  \( S_B > S > S_C > S_{CB} > S_{AC} > S_{ABC} > S_{AB} > S_A \)
Continuous improvement

- Given the team’s preferences:
  - Jan: $S_{AB} > S_A > S > S_B > S_C > S_{AC} > S_{BC} > S_{ABC}$
  - Pat: $S_A > S > S_B > S_C > S_{BC} > S_{AC} > S_{ABC} > S_{AB}$
  - Michael: $S_B > S > S_C > S_{CB} > S_{AC} > S_{ABC} > S_{AB} > S_A$

- $S_{ABC} > S_{AB} > S_A > S$
  - The team thus moves from $S$ to $S_A$ to $S_{AB}$ to $S_{ABC}$
  - But everyone agrees that $S > S_{ABC}$

- Continuous improvement made the product worse
A tenure case

- Three assistant professors get student ratings
  - Marc > Raj > Jan: 33 students
  - Marc > Jan > Raj: 0 students
  - Raj > Marc > Jan: 25 students
  - Raj > Jan > Marc: 14 students
  - Jan > Marc > Raj: 25 students
  - Jan > Raj > Marc: 17 students

- Scores (pick one): Jan 42, Raj 39, Marc 33
- Jan gets tenure
But Raj accepts another job

- Ratings for Marc and Jan only
  - Marc>Jan 33 students
  - Marc>Jan 0 students
  - Marc>Jan 25 students
  - Jan>Marc 14 students
  - Jan>Marc 25 students
  - Jan>Marc 17 students

- Scores (pick one): Jan 56, Marc 58
- Who should get tenure?
Market survey

Two new products, A and B are tested in two markets against the existing product.

San Francisco
A: 8/10 (80%)
B: 18/25 (72%)

New York
A: 10/30 (33%)
B: 3/15 (20%)

A beats B in both cities
Market survey

We aggregate the results:

San Francisco
A: 8/10 (80%)
B: 18/25 (72%)

New York
A: 10/30 (33%)
B: 3/15 (20%)

A is preferred 18 out of 40 times
B is preferred 21 out of 40 times
B is the better design
Loaded dice

- A game of chance: there are 3 dice, each with 3 numbers on the sides (the numbers repeat on opposite sides)
  - $D_A$: 1, 6, 8
  - $D_B$: 3, 5, 7
  - $D_C$: 2, 4, 9
- You choose a die, then I choose a die. We roll them and the person whose die lands with the highest number wins
You lose!

A beats B
B beats C
C beats A

<table>
<thead>
<tr>
<th></th>
<th>A vs. B</th>
<th>B vs. C</th>
<th>C vs. A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>
Black matter and such

- Engineering faux pas
  - Data fusion
  - Multi-scale modeling
  - Multi-objective optimization
  - QFD, Pugh, AHP

- Black matter—D. Saari suggests that there is much less (90% less) black matter than current models predict
So what?

- Seemingly good models might not be so good after all
- When rationality runs out, all bets are off—we go into a mode of damage control
- We lose the ability to do rational design at about the time we assign the second engineer to the project
- Really bad things can happen from here
- Is there hope anywhere?
  - Decision theory and Bayes theorem
  - Modeling
  - Mechanism design
Recommendations

- We need to take note of the mathematics of systems engineering
- Let’s take engineering education away from the scientists and start teaching engineering (decision making vs. problem solving)
- Stop teaching engineering as a deterministic application of scientific principles
- Stop teaching engineering students as though every one will become a professor
- Teach judgment